Section 4.3 New Rules for Derivatives

"Product and Quotient Rules for Derivatives"

Product Rule

Example: $(f(x) \cdot g(x))' = f'(x) * g(x) + g'(x) * f(x)$

or

$(fg)' = f'g + g'f$

Quotient Rule $\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$

Product means multiply, thus Example $h(x) = \underbrace{(x+3)}_{f}\underbrace{(3x-7)}_{g}$

Step 1 Label your f and g.

Step 2 $f = x+3$ $\quad g = 3x-7$

$f' = 1$ $\quad g' = 3$

Step 3 $h'(x) = 3(x+3) + 1(3x-7)$

(Lines from above)

Step 4 Clean it up! $3(x+3) + 1(3x-7)$

$3x+9+3x-7$

$h'(x) \boxed{= 6x+2}$

Another way: $h(x) = 3x^2 - 7x + 9x - 21$

$= 3x^2 + 2x - 21$

$h'(x) = 6x+2$

*Professor's Golden Rule

Always try to simplify the algebra first

Example $\underbrace{(x+3)^{12}\cdot(x-1)^{15}}$ $= f(x)$

Reason why product & quotient rule created. B/C this would be very difficult to solve algebraicall.

$f=(x+3)^{12}$ $g=(x-1)^{15}$

$\boxed{\text{Step 1}}$ $f'=12\,\underline{(x+3)}^{11}$ $g'=15(x-1)^{14}$

always multiply by its derivative
$\therefore x+3 =$ derivative of 1

$\boxed{\text{Step 2}}$ $f'=12(x+3)^{11}(1)$ $g'=15(x-1)^{14}(1)$

$\boxed{\text{Step 3}}$ Clean up!!

$f'=12(x+3)$ $g'=15(x-1)^{14}$

$\boxed{\text{Step 4}}$ $f=(x+3)^{12}$ $g=(x-1)^{15}$

$f'=12(x+3)^{11}$ $g'=15(x-1)^{14}$

Cross multiply

$\boxed{\text{Step 5}}$ $= \underset{(f')}{12(x+3)^{11}}\ \underset{(g)}{(x-1)^{15}} + \underset{(g')}{15(x-1)^{14}}\ \underset{(f)}{(x+3)^{12}}$

$\boxed{\text{Step 6}}$ Clean up!! $= \underbrace{3(x+3)(x-1)}_{\text{Base powers}}$

Regular Formula to clean up $12\cancel{(x+3)}^{11}(x-1)^{15} + 15\cancel{(x-1)}^{14}(x+3)^{12}$

$= 3(x+3)^{11}(x-1)^{14}\,[4(x-1) + 5(x+3)]$

$= (x-1)^{15}$ and $(x+3)^{12}$ left @ top

Cont: $3(x+3)^{11}(x-1)^{14}\ [4x-4+5x+15]$

$9x+11$

$\boxed{=3(x+3)^{11}(x-1)^{14}(9x+11)}$

Example: $f(x)=(3x+4)^{18}$

Step 1 $f'(x)=18(3x+4)^{17}$ multiply using power-rule & subtract

Step 2 Find derivative of $3x+4$

$3x+4 = 3$ derivative

Step 3 $f'(x)=18(3x+4)^{17}(3)$

$\boxed{=54(3x+4)^{17}}$ Example of Chain Rule Section 4.4

~~Step 4~~

Example: $2x^3y^2-4x^5y^8$

Step 1 $2x^3y^2[1-2x^2y^6]$ Step 2 → (Plug in what is left over)

Example $f(x)=(5x^2+2x-1)^{14}$

Step 1 $14(5x^2+2x-1)^{13}$

Step 2 $14(5x^2+2x-1)^{13}$ Find derivative Of $(5x^2+2x-1)$

$=5(2)x^{2-1}+2(1)x^{1-1} = 10x+2$

Step 3 $28(5x^2+2x-1)^{13}(5x+1)$ Common factor $2(5x+1)$

Chain & Power Rule combined

$h(x) = (3x+2)^{11}(5x-1)^{13}$

(f) (g)

$\boxed{\text{Step 1}}$ $11(3x+2)^{10}$ $13(5x-1)^{12}$

Power Rule $\boxed{\text{Step 2}}$ $11(3x+2)^{10}$ $13(5x-1)^{12}$

Derivative Derivative

$3(1)x^{1-1} = 3$ $5(1)x^{1-1} = 5$

Chain Rule $\boxed{\text{Step 3}}$ $33(3x+2)^{10} = f'$ $65(5x-1)^{12} = g'$

$(3x+2) = f$ $(5x-1) = g$

$\boxed{\text{Step 4}}$ $33\cancel{(3x+2)^{10}}(5x-1)^{13} + 65\cancel{(5x-1)^{12}}(3x+2)^{11}$

$\underbrace{(3x+2)^{10}(5x-1)^{12}}_{\text{Common Factors}}\ \underbrace{[33(5x-1)+65(3x+2)]}_{\text{what is left over in above equation less common factors}}$

*If done correctly the info within the bracket will only have ( ) at one power.

$\boxed{\text{Step 5}}$ Clean up brackets $[33(5x-1)+65(3x+2)]$

$= [165x-33+195x+130]$

$= [360x+97]$

$\boxed{\text{Step 6}}$ $(3x+2)^{10}(5x-1)^{12}(360x+97)$